Nonlocality – A Review of Some Recent Work

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Abstract: Discussions of nonlocality in QM are usually associated with the Bell inequality and its well-attested experimental violation. Less well-known are attempts to give purely algebraic proofs of nonlocality. The somewhat tangled history of these attempts is outlined and some novel philosophical conclusions are drawn concerning possible interpretations of nonlocality.

Introduction: Ever since its inception it has been felt that QM would be much less mystifying, much more comprehensible if all observables at all times possessed sharp values which were simply discovered by measurement. The hidden-variable programme in QM has pursued this possibility in spite of many 'proofs' in the literature that it cannot be done.

There are 2 main ~~difficulties~~ questions

1) Can we consistently assign values to all observables?

2) Can we recover the right statistical predictions?

Under certain plausible assumptions the answer to both questions is NO.

1.) If we assume that functional relationships between compatible (commuting) observables are preserved under value assignments eg. $[Q^2] = ([Q])^2$ and so on

Then we get a purely algebraic contradiction known as the Kochen–Specker Paradox (1967) — although implicit in the work of Gleason (1957).

2.) Under the assumption of locality that if I snap my fingers the door over there does not open mysteriously and instantaneously then one cannot get the statistics right in certain correlation experiments involving two widely separated systems such as two photons or two protons moving away from each other in opposite directions. More specifically one can derive a relation between correlation coefficients called the Bell inequality which is violated by the predictions of QM under appropriate conditions. — most famously in the very recent Aspect experiment (Dec. 1982)

Bell (1964)

However it has been objected to the Bell inequality approach to nonlocality that other assumptions concerning the machinery of probability theory and its application to QM are involved in the derivation of the Bell inequality, and the violation of the Bell inequality might be attributed to shortcomings here, rather than to nonlocality.

Interest has therefore arisen in trying to give proofs of nonlocality by purely algebraic methods similar to those employed in the Kochen-Specker paradox. This idea was specifically advocated by Jeffrey Bub in 1976. The approach envisaged by Bub actually turned out to be a dead-end as was finally shown by Bill Demopoulos in 1980. At this moment myself and Peter Heywood tried a rather different approach, that had been canvassed originally by Kochen in unpublished work and by Allen Stairs in his 1978 Ph.D. thesis in a rather different context to discussion of nonlocality. Redhead & Heywood's result published in Foundations of Physics in May 1983.

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CoR: (Correspondence Rule):
There is a 1:1 correspondence between self-adjoint operators and observables

$$\hat{Q} \leftrightarrow Q.$$

R (Reality Principle):

If there is an operationally defined number associated with $\hat{Q}$ (i.e. distributed probabilistically according to the statistical algorithm of QM for $\hat{Q}$) then $\exists$ element of reality (an observable) associated with that number and measured by it (assuming FM).

Kochen-Specker Paradox

Sharp Value + FUNC → Contradiction

Show ⑬ Statement of FUNC.

Proof of K-S contradiction

Show ⑫, ④, ⑤ ⑥ and ⑰

Hughes presentation in QM.
Spin 1 example
2-colour theorem
— 2-dimensional case
— Belinfante's proof.

Break FUNC by decontextualization (Van Fraassen 1973)

Show (13a) Distinguish A_B, A_C

But assume FUNC* $A_B = A_C$ if B, C commute

Proof of FUNC

* $\underline{A} + \underline{\text{Corr.}} + \underline{R} \Rightarrow$ FUNC.

Measure $[Q]$ Form $f([Q])$

— Distributed probabilistically according to $f(\hat{Q})$

By R ∴ ∃ observable measured by $f([Q])$ associated with operator $f(\hat{Q})$

By Corr There is only one such observable — denote by $f(Q)$

Hence $[f(Q)] = f([Q])$

Contextuality

Slide (14)
Define $[AB]^{\phi}$ — ontological contextuality
and $[AB]^{\phi}(C)$ — Environmental contextuality

Locality Principle

OLOC.

$H_1 \otimes H_2$.

measured → A extends to
$A \otimes 1$ ← non measured

Slide (14b) locally measured observables like $A \otimes I$ are not split by ontological contextuality

Slide (14c) ELOC as in Bell's theorem

N.B. 1) ELOC only if locality principle of OLOC is satisfied
2) In terms of measurement results
OLOC = ELOC.

Redhead–Heywood Result

Func^{n} + VR + ELOC + OLOC ⇒ Contradiction

Slide (16) VR $\rho_{Q}^{\phi}(\lambda) = 0 \Rightarrow [Q]^{\phi} \neq \lambda$

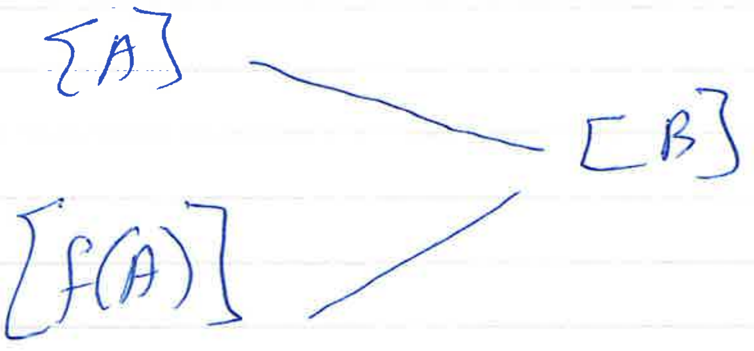
Fine (1974): 'Hidden variables are no story, hidden assumptions matter'

Sketch Proof

Step 1 Step 2

1 2

Ⓑ Ⓑ



Hence establish a relation between [A] and [f(A)] of form

$$[f(A)] = f([A])$$

$\Longrightarrow$ K-S paradox on system ①

Conclusion

In fact the result established is not quite FUNC, but one is still able to derive a contradiction.

Violating OLOC ontological holism

Violating FLOC Similar to non-locality in the Bell argument.

Back to history

Could one show LOC violated directly. Bub (1976)?

Maczynski (1971) showed no contradiction for maximal bundles only.

Could one prove that Maczynski's theorem cannot be extended to maximal and locally maximal observables?

Demopoulos first claimed M's theorem could not be extended.

His proof was criticised by Bub and Paul Humphreys.

So then Demopoulos proved M's theorem could be extended (1980)

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So no purely algebraic proof of nonlocality but the Redhead-Heywood result comes very close to it only involves probability via 'Value' Rule

So in this sense it may be regarded as superior to the Bell-type proofs.